3D seismic survey design by maximizing the spectral gap

Yijun Zhang[1,], Ziyi Yin[1,*], Oscar López[2], Ali Siahkoohi[3], Mathias Louboutin[1], and Felix J. Herrmann[1]*
[1] *Georgia Institute of Technology*
[2] *Florida Atlantic University*
[3] *Rice University*
[*] *contributed equally*

SUMMARY

The massive cost of 3D acquisition calls for methods to reduce the number of receivers by designing optimal receiver sampling masks. Recent studies on 2D seismic showed that maximizing the spectral gap of the subsampling mask leads to better wavefield reconstruction results. We enrich the current study by proposing a simulation-free method to generate optimal 3D acquisition by maximizing the spectral gap of the subsampling mask via a simulated annealing algorithm. Numerical experiments confirm improvement of the proposed method over receiver sampling locations obtained by jittered sampling.

INTRODUCTION

Acquiring fully sampled seismic data is expensive in practice. Thanks to recent advancements in compressive sensing, seismic data can be randomly sampled along spatial coordinates to improve acquisition efficiency (Candès et al., 2006; Mosher et al., 2014; Kumar et al., 2015; Chiu, 2019). The subsampled data can be consequently processed by wavefield reconstruction techniques to recover the fully sampled data (Hennenfent and Herrmann, 2008; Kumar et al., 2015; Zhang et al., 2020). While uniform random sampling (Candès and Tao, 2010; Candes and Recht, 2012) and jittered subsampling schemes, which control the maximum gap size in subsampled data (Herrmann and Hennenfent, 2008), are easy to generate, they may not be optimal and can be improved through the resolution of specific optimization problems (Mosher et al., 2014; Manohar et al., 2018). Notably, a mutual coherence-based global optimization scheme, initially proposed by Mosher et al. (2014), Li et al. (2017), and later expanded upon by Titova et al. (2019), has been developed to enhance reconstruction quality. Mosher et al. (2014) proposed a simulation-based acquisition design method based on the advancements in compressive sensing to search for optimized sampling schemes. Similarly, Guo and Sacchi (2020) optimized time-lapse seismic acquisition using prior seismic data (Manohar et al., 2018). Guo et al. (2023) also proposed a simulation-based survey design method based on reinforcement learning. These methods have shown promising results, but determining the optimal source-receiver layout through combined wavefield simulations and recoveries is computationally expensive and requires detailed information of the seismic data, making it infeasible for optimizing for source and receiver locations in a large-scale 3D survey.

Matrix completion is a computationally efficient technique used to reconstruct fully sampled data from sparsely sampled seismic data (Kumar et al., 2015). In matrix completion theory, the spectral gap, which measures the connectivity of the graph in expander graph theory, is employed to predict and partially quantify the quality of the matrix completion result solely based on the binary subsampling mask (Bhojanapalli and Jain, 2014). López et al. (2023) further confirms that the success of seismic wavefield reconstruction through universal matrix completion can be predicted based on the ratio of the first two singular values of the binary subsampling masks.

While recent work by López et al. (2023) eliminates the need for multiple expensive wavefield reconstructions with different sampling mask candidates, it does not yet provide a method to generate sampling masks that maximize the spectral gap. Zhang et al. (2022) introduced a practical algorithm utilizing simulated annealing (Henderson et al., 2003) to generate acquisition geometries with small spectral gap ratios, and this algorithm was further extended to time-lapse seismic acquisition by Zhang et al. (2023). However, the application of this technique to 3D seismic cases has not been explored yet. In this expanded abstract, we will introduce a practical algorithm aimed specifically at minimizing the spectral gap ratio of the binary subsampling mask corresponding to a 3D seismic survey.

We organize this expanded abstract as follows. First, we present the proposed optimization problem aimed at maximizing the spectral gap ratio of 3D subsampling masks. Next, we introduce an intriguing property of spectral gap ratio of the binary masks for the unique case of 3D seismic surveys. Thanks to this property, we are able to reduce the computational cost of our algorithm. Finally, we demonstrate numerical experiments conducted on the synthetic 3D Compass dataset (Jones et al., 2012) and showcase an enhancement in recovery quality by juxtaposing two wavefield reconstruction results where the receiver locations are obtained by the jittered subsampling method (Hennenfent and Herrmann, 2008) and our proposed method.

METHODOLOGY

The quality of seismic wavefield reconstruction through universal matrix completion (Bhojanapalli and Jain, 2014) can be predicted by evaluating the ratio between the first two singular values of binary sampling masks, denoted as $\sigma_2(\mathbf{M})/\sigma_1(\mathbf{M}) \in [0,1]$, where the binary matrix $\mathbf{M}$ contains 1's to indicate sampled data and 0's otherwise. Termed as the spectral gap ratio, this measure offers a cost-effective means to quantitatively assess recovery quality. A smaller spectral gap ratio implies enhanced connectivity within the graphs formed by binary sam-

10.1190/image2023-3895546.1

pling masks, resulting in improved wavefield recovery (López et al., 2023). The spectral gap ratio has proven effective in predicting and enhancing 2D wavefield reconstruction performance (Zhang et al., 2022) and has been successfully applied to time-lapse survey design (Zhang et al., 2023). In this expanded abstract, we specifically consider 3D survey design where receivers are missing and sources are fully sampled. We propose a cheap algorithm based on SGR minimization to optimize sparse geometries for 3D seismic acquisition.

Optimized 3D seismic acquisition

3D wavefield reconstruction based on low-rank matrix completion relies on the non-canonical Source-X/Receiver-X (columns) Source-Y/Receiver-Y (rows) organization of the data into a matrix in order to leverage the inherent low-rank characteristics of seismic data (Kumar et al., 2015). Following this success, we aim to minimize the SGR of the subsampling mask in the same domain. In this scheme, the subsampling mask is represented as $\mathbf{M} \in \{0,1\}^{(N_{sx}\times N_{rx})\times(N_{sy}\times N_{ry})}$, where N_{sx} and N_{sy} represent the number of sources along the x and y coordinates, respectively, and N_{rx} and N_{ry} represent the number of receivers along these coordinates. Motivated by the success on (time-lapse) 2D seismic survey design reported by Zhang et al. (2022) and Zhang et al. (2023), we solve the following optimization problem to minimize the spectral gap ratio of the 3D acquisition mask:

$$\underset{\mathbf{M}}{\text{minimize}} \quad \sigma_2(\mathbf{M})/\sigma_1(\mathbf{M}) \quad \text{subject to} \quad \mathbf{M} \in \mathscr{C}. \tag{1}$$

The objective function is the SGR of the subsampling mask, where σ_1 and σ_2 represent the first and the second singular values, respectively. In order to ensure the feasibility of optimized binary masks with a receiver subsampling ratio denoted as $\rho \in (0,1)$, we incorporate a constraint denoted as $\mathscr{C} = \bigcap_{i=1}^{4} \mathscr{C}_i$. This constraint encompasses four components: a cardinality constraint defined as

$$\mathscr{C}_1 = \{\mathbf{M} \mid \#(\mathbf{M}) = \lfloor N_{rx} \times N_{ry} \times \rho \rfloor \times N_{sx} \times N_{sy}\}, \tag{2}$$

a binary mask constraint as

$$\mathscr{C}_2 = \{\mathbf{M} \mid \mathbf{M} \in \{0,1\}^{(N_{sx}\times N_{rx})\times(N_{sy}\times N_{ry})}\}, \tag{3}$$

and constraints, $\mathscr{C}_3$ and $\mathscr{C}_4$, to enforce lower bounds for the number of subsampling points for each row and column of the binary subsampling matrix, respectively, in order to avoid missing row or column in the matrix. They are defined as

$$\begin{gathered}\mathscr{C}_3 = \{\mathbf{M} \mid \#(\mathbf{M}_i) \geq m\} \quad \text{and} \quad \mathscr{C}_4 = \{\mathbf{M} \mid \#(\mathbf{M}^j) \geq n\} \\ \text{where} \quad i = 1, \cdots, N_{sx} \times N_{rx} \quad \text{and} \quad j = 1, \cdots, N_{sy} \times N_{ry}.\end{gathered} \tag{4}$$

Here, $\mathbf{M}_i$ and $\mathbf{M}^j$ represent the i-th row and the j-th column of matrix $\mathbf{M}$, respectively. m represents the lower bound for number of subsampling points for each row, and n for each column. These lower-bound constraints ensure that the binary subsampling matrix does not contain any empty rows or columns. This is important because having an empty row or column can be detrimental to the process of matrix completion.

An intriguing property of the spectral gap ratio

Despite the fact that this proposed optimization could be solved using simulated annealing (Zhang et al., 2022, 2023), the binary subsampling matrix $\mathbf{M}$ can still be large for 3D seismic cases, which potentially slows down the algorithm in practice. To overcome this problem, we have fortunately discovered that when sources are fully sampled, each single-receiver block of the global sampling matrix is either fully sampled or empty depending on whether that specific receiver is sampled. Consequently, the block structure of the global matrix leads to the exact same singular values as a single-source receiver sampling mask, as shown in Figure 1. We can therefore optimize a single-source mask to obtain the global optimized mask. The main computational cost therein is computing the first two singular values of the receiver sampling mask, which is negligible compared to approaches that require wave simulations (Mosher et al., 2014; Guo et al., 2023). The resulting optimal mask with the lowest SGR indicates the receiver sampling locations that favor 3D wavefield reconstruction via matrix completion in the non-canonical organization domain.

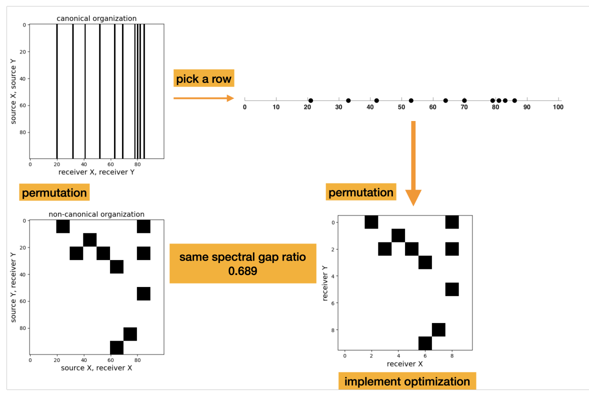

Figure 1: Spectral gap ratio of the data matrix in the non-canonical Source-X/Receiver-X (columns) Source-Y/Receiver-Y (rows) domain is the same as the spectral gap ratio of the single-source receiver sampling matrix.

Experiment

We illustrate the efficacy of our method via a numerical experiment on a simulated 3D marine dataset over the compass model (Jones et al., 2012). The data volume consists of 501 time samples, 1681 sources and $10,000$ receivers. The distance between the adjacent sources and receivers are 150m and 25m in each direction, respectively, with a time sampling interval of 0.01s. By using jittered subsampling (Herrmann, 2010), we removed 90% of the receivers. This results in a binary matrix with a SGR of 0.507 in the non-canonical domain. After applying simulated annealing algorithm, the SGR of the mask effectively decreases to 0.328. To validate the efficacy of our acquisition design method, we perform data reconstruction on a frequency slice at 16.8Hz via weighted matrix completion (Zhang et al., 2020) for the two subsampled datasets with jittered subsampling mask and the optimized mask. This data reconstruction process was performed on each dataset individually. The reconstruction results are shown in Figure 2, providing evidence for the efficacy of the acquisition design strategy employed throughout the study. The reconstruction signal-to-noise ratio (SNR) obtained from data observed at jittered sampled receiver locations is $10.88dB$, which is lower than the reconstruction SNR obtained from data observed at specified receiver locations obtained by solving Equation 1 ($12.27dB$). This distinction is crucial, with an increase of approximately $1.4dB$ in SNR. This finding supports the hypothesis that the optimized receiver positions can lead to a superior seismic survey, ultimately improving the performance of the wavefield reconstruction. Specifically, this algorithm is computationally inexpensive compared to simulation-based survey design methods, because each iteration of this algorithm only needs to calculate the first two singular values of the receiver sampling mask — a small 100×100 matrix.

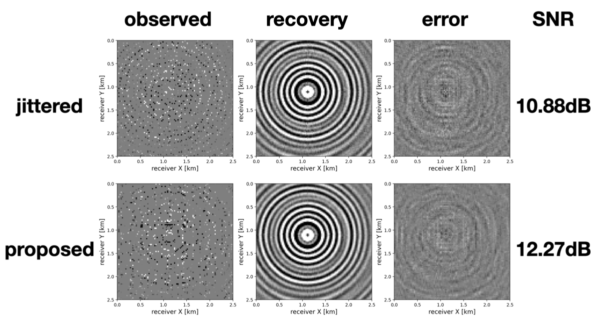

Figure 2: Comparison of data reconstruction performance for receiver locations sampled by the jittered method and the proposed method. There is about $1.4dB$ SNR improvement.

CONCLUSIONS

Our expanded abstract presents the first numerical case study that applies spectral gap ratio minimization techniques for 3D seismic acquisition design. Rather than requiring costly wave simulations, the proposed method only relies on a single binary matrix optimization which is computationally inexpensive. Through a representative numerical experiment conducted on 3D Compass dataset, we conclude that our proposed method yields an optimal subsampling mask that is highly suitable for 3D wavefield reconstruction based on matrix completion. This cheap while effective optimization scheme has the potential to scale to industry-size 3D survey design problems.

ACKNOWLEDGEMENT

This research was carried out with the support of Georgia Research Alliance and partners of the ML4Seismic Center.

REFERENCES

Bhojanapalli, S., and P. Jain, 2014, Universal matrix completion: International Conference on Machine Learning, PMLR, 1881–1889.

Candes, E. J., and B. Recht, 2012, Exact matrix completion via convex optimization: Communications of the ACM, **55**, 111–119, doi: https://doi.org/10.48550/arXiv.0805.4471.

Candès, E. J., J. Romberg, and T. Tao, 2006, Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information: IEEE Transactions on Information Theory, **52**, 489–509, doi: https://doi.org/10.1109/TIT.2005.862083.

Candès, E. J., and T. Tao, 2010, The power of convex relaxation: Near-optimal matrix completion: IEEE Transactions on Information Theory, **56**, 2053–2080, doi: https://doi.org/10.48550/arXiv.0903.1476.

Chiu, S. K., 2019, 3D attenuation of aliased ground roll on randomly undersampled data: 89th Annual International Meeting, SEG, Expanded Abstracts, 4560–4564, doi: https://doi.org/10.1190/segam2019-3208741.1.

Guo, Y., R. Lin, and M. D. Sacchi, 2023, Optimal seismic sensor placement based on reinforcement learning approach: An example of OBN acquisition design: IEEE Transactions on Geoscience and Remote Sensing, **61**, 1–12, doi: https://doi.org/10.1109/TGRS.2023.3247593.

Guo, Y., and M. D. Sacchi, 2020, Data-driven time-lapse acquisition design via optimal receiver-source placement and reconstruction: 90th Annual International Meeting, SEG, Expanded Abstracts, 66–70, doi: https://doi.org/10.1190/segam2020-3426928.1.

Henderson, D., S. H. Jacobson, and A. W. Johnson, 2003, The theory and practice of simulated annealing: Springer, 287–319.

Hennenfent, G., and F. J. Herrmann, 2008, Simply denoise: Wavefield reconstruction via jittered undersampling: Geophysics, **73**, no. 3, V19–V28, doi: https://doi.org/10.1190/1.2841038.

Herrmann, F. J., 2010, Randomized sampling and sparsity: Getting more information from fewer samples: Geophysics, **75**, no. 6, WB173–WB187, doi: https://doi.org/10.1190/1.3506147.

Herrmann, F. J., and G. Hennenfent, 2008, Non-parametric seismic data recovery with curvelet frames: Geophysical Journal International, **173**, 233–248, doi: https://doi.org/10.1111/j.1365-246X.2007.03698.x.

Jones, C., J. Edgar, J. Selvage, and H. Crook, 2012, Building complex synthetic models to evaluate acquisition geometries and velocity inversion technologies: 80th EAGE Conference and Exhibition 2018, cp–293.

Kumar, R., C. Da Silva, O. Akalin, A. Y. Aravkin, H. Mansour, B. Recht, and F. J. Herrmann, 2015, Efficient matrix completion for seismic data reconstruction: Geophysics, **80**, no. 5, V97–V114, doi: https://doi.org/10.1190/geo2014-0369.1.

Li, C., S. T. Kaplan, C. C. Mosher, J. D. Brewer, and R. G. Keys, 2017, Compressive sensing: U.S. Patent 9,63,193.

López, O., R. Kumar, N. Moldoveanu, and F. J. Herrmann, 2023, Spectral gap-based seismic survey design: IEEE Transactions on Geoscience and Remote Sensing, **61**, doi: https://doi.org/10.1109/TGRS.2023.3237464.

Manohar, K., B. W. Brunton, J. N. Kutz, and S. L. Brunton, 2018, Data-driven sparse sensor placement for reconstruction: Demonstrating the benefits of exploiting known patterns: IEEE Control Systems Magazine, **38**, 63–86, doi: https://doi.org/10.1109/MCS.2018.2810460.

Mosher, C., C. Li, L. Morley, Y. Ji, F. Janiszewski, R. Olson, and J. Brewer, 2014, Increasing the efficiency of seismic data acquisition via compressive sensing: The Leading Edge, **33**, no. 4, 386–391, doi: https://doi.org/https://doi.org/10.1190/tle33040386.1.

Titova, A., M. B. Wakin, and A. Tura, 2019, Mutual coherence in compressive sensing seismic acquisition: 89th Annual International Meeting, SEG, Expanded Abstracts, 122–126, doi: https://doi.org/10.1190/segam2019-3203905.1.

Zhang, Y., M. Louboutin, A. Siahkoohi, Z. Yin, R. Kumar, and F. J. Herrmann, 2022, A simulation-free seismic survey design by maximizing the spectral gap: Second International Meeting for Applied Geoscience & Energy, SEG/AAPG, Expanded Abstracts, 15–20, doi: https://doi.org/10.1190/image2022-3751690.1.

Zhang, Y., S. Sharan, O. Lopez, and F. J. Herrmann, 2020, Wavefield recovery with limited-subspace weighted matrix factorizations: 90th Annual International Meeting, SEG, Expanded Abstracts, 2858–2862, doi: https://doi.org/10.1190/segam2020-3428365.1.

Zhang, Y., Z. Yin, O. Lopez, A. Siahkoohi, M. Louboutin, R. Kumar, and F. J. Herrmann, 2023, Optimized time-lapse acquisition design via spectral gap ratio minimization: Geophysics, **88**, no. 4, A19–A23, doi: https://doi.org/10.1190/geo2023-0024.1.